1. Commutative Transformations (6 points)

Which 3D transformations are commutative? Circle yes or no. (1 point each)

a) Translation followed by a translation       Yes  No
b) Rotation followed by a uniform scale       Yes  No
c) Rotation followed by a non-uniform scale   Yes  No
d) Rotation followed by a translation         Yes  No
e) Scale followed by a scale                  Yes  No
f) Translation followed by a shear            Yes  No
2. 2D Transformations (10 points)

a) Draw the result of the following 2D transformations on the graph below, where the thick lines indicate the 0 coordinates in x and y:

a.1) A unit square translated x=2, y=3 followed by a uniform scale of 3. (3 points)

a.2) A unit square translated x=8, y=2, then rotated theta=45 degrees counterclockwise. (3 points)

b) Give the 2D transformation matrices for the transformations above (two matrices expected for each of a.1 and a.2). (4 points)
3. Matrices (10 points)

Label each of the following 4x4 matrices with what they are. The options are: scale, orthogonal projection, perspective projection, translation, rotation. Hint: there is one matrix of each type. (2 points each)

a) 
\[
\begin{bmatrix}
1 & 0 & 0 & 12 \\
0 & 1 & 0 & -5 \\
0 & 0 & 1 & 8.5 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

b) 
\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0.4 & 0.91 & 0 \\
0 & -0.91 & 0.4 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

c) 
\[
\begin{bmatrix}
0.4 & 0 & 0 & 0 \\
0 & 0.4 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

d) 
\[
\begin{bmatrix}
5 & 0 & 0 & 0 \\
0 & 5 & 0 & 0 \\
0 & 0 & 5 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

e) 
\[
\begin{bmatrix}
-1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & -11 & 10 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]
4. Cameras (8 points)

Two cameras are said to converge on a point if the point lies along the optical axes of both cameras. Let C1 be a camera with a focal point $e_1$ of (0,0,0), an up vector $up_1$ of (0,1,0) and a camera viewing direction $ed_1$ of (a, 0, c). Let C2 be another camera with a focal point $e_2$ of (b, 0, 0), an up vector $up_2$ of (0,1,0) and a camera viewing direction $ed_2$ of (-a, 0, c). What is the 3D position of the point on which the two cameras converge?

5. Canonical View Volume (8 points)

a) What is the canonical view volume? Either draw it labeled or describe it. (4 points)

b) Give the transformation matrix which transforms the canonical view volume to a 256x512 pixel window. (4 points)
6. Graphics Pipeline (10 points)

a. Camera Space
b. Image Space
c. Inner Space
d. Canonical View Space
e. Object Space
f. Outer Space
g. Projective Space
h. World Space

Select the correct spaces (=coordinate systems) from above and list them in the order that we expect each vertex to go through in the traditional transformation process:

Fill in letters here: 1. ____ 2. _____ 3. _____ 4. _____ 5. _____

7. Lighting (8 points)

Assume that there is a point light at position p that is illuminating a point v on a surface. Explain why the intensity of the light is proportional to $1/||p-v||^2$, i.e., one over the squared distance between the light source and the point on the surface. (3-4 sentences)
8. Rasterizing a Circle (10 points)

You are tasked with developing an algorithm which rasterizes a circle with radius r. Start at pixel location $x=-r$, $y=0$. Assume you are rasterizing in clockwise direction. Sketch this scenario along with what choices should be made for the next pixel. Include pseudo-code in your answer.

9. Rasterization Artifacts (10 points)

a) Why does linear interpolation of texture coordinates in screen space lead to artifacts? Explain using a sketch and 2-3 explanatory sentences. (5 points)

b) What technique can be used to correct these artifacts? Explain its basic idea in 2-3 sentences. (5 points)
10. Texture Mapping and Mipmapping (10 points)

a) How many texel values have to be read to perform nearest neighbor texture filtering, bilinear texture filtering, and trilinear mipmapping? (2 points)

b) In which order are the texel values in each of the three methods above being averaged? (2 points)

c) What is the advantage of trilinear mipmapping over bilinear filtering (2-3 sentences)? (3 points)

d) Explain why trilinear mipmapping is subject to a trade-off between blurriness and aliasing. What could be done to achieve better antialiasing than trilinear mipmapping? (3-4 sentences) (3 points)
11. Advanced Texture Mapping (10 points)

a) Describe the basic idea of bump mapping in 2-3 sentences. (2 points)

b) Name one advantage and one disadvantage of bump mapping compared to creating geometry (e.g., a detailed triangle mesh) to achieve the realistic appearance of a surface which is not smooth. (2 points)

c) Name one major difference between Bump Mapping and Displacement Mapping. (2 points)

d) Explain in 1-2 sentences what Multi-Texturing is. (2 points)

e) Name two examples for what Multi-Texturing can be used for. (2 points)