1. Points and Vectors (10 points)

a) Compute the dot product of vectors \( a = (6, 8, 4) \) and \( b = (9, 12, 6) \). What is the angle between \( a \) and \( b \)? (4 points)

b) Given two vectors \( b \) and \( c \) and their cross product \( a = b \times c \). What is the cross product \( c \times b, b \times a, \) and \( a \times a \)? (6 points)
2. Colors (10 points)

a) What does a Chromaticity Diagram visualize? What characterizes the colors on its boundary? How can this diagram be used to show the color gamut of a monitor? (6 points)

b) What is the main property of a perceptually uniform color space? Name a common perceptually uniform color space. (4 points)

3. Shaders (10 points)

a) List one advantage and one disadvantage of computing shading on a per pixel basis compared to shading on a per vertex basis. (2 points)

b) Write down pseudo-code for a vertex and fragment shader that perform per pixel diffuse shading for a single light source. The input to the vertex shader is the vertex position, normal, and diffuse color, and the light position and color. You can use your own names for these variables, there is no need to use the OpenGL names. (8 points)
4. CUDA (10 points)

a) What is the purpose of CUDA? (2 points)

b) Name three fundamental differences between CUDA and GLSL, and explain in one sentence for each of them. (6 points)

c) Assume that you are tasked with parallelizing a piece of code in the C programming language. Name two fundamental differences between parallelizing it for the CUDA architecture and parallelizing it for a multi-core CPU. (2 points)

5. View Frustum Culling (10 points)

a) Explain how per-object view frustum culling works, and what its main benefit is. Give an example of a scenario where it is particularly beneficial. (2 points)

b) Given a perspective view frustum bounded by four planes. All planes go through the origin. Their outward pointing normals are:

\[
\begin{bmatrix}
-1/\sqrt{2} \\
1/\sqrt{2} \\
0
\end{bmatrix}, \begin{bmatrix}
-1/\sqrt{2} \\
-1/\sqrt{2} \\
0
\end{bmatrix}, \begin{bmatrix}
-1/\sqrt{2} \\
0 \\
1/\sqrt{2}
\end{bmatrix}, \begin{bmatrix}
-1/\sqrt{2} \\
0 \\
-1/\sqrt{2}
\end{bmatrix}
\]

Does the sphere centered at (1, 2, 3) with radius 1 intersect this view frustum? (Derivation required; 8 points)
6. Bezier Curve (10 points)

Given a cubic Bezier curve \( x(t) \), which is defined by control points \( p_0, p_1, p_2, p_3 \). Derive the equation \( x'(t) \) for the tangent of the curve at each value of \( t \). (8 points) Evaluate \( x'(0) \) and \( x'(1) \). (2 points)

Remember that the cubic Bernstein polynomials are:

\[
\begin{align*}
B_0(t) &= (1 - t)^3 \\
B_1(t) &= 3t (1 - t)^2 \\
B_2(t) &= 3t^2 (1 - t) \\
B_3(t) &= t^3
\end{align*}
\]

7. Environment Mapping (10 points)

a) What is an environment map? (2 points)

b) Name two ways to create environment maps. (2 points)

c) Name two advantages of cube maps over spherical environment maps. (2 points)

d) Is it computationally more expensive to compute shading of a diffuse surface or a specular surface when an environment map is used? Why? (2 points)

e) Name a technique to speed up the rendering of diffuse surfaces with environment maps. Explain its basic idea in one sentence. (2 points)
8. Toon Shading (10 points)

a) What is the goal of toon shading? (2 points)

b) How can toon shading be accomplished in real-time? (2 points)

c) Explain how silhouette edges can be detected for toon shading. (4 points)

d) Name two parameters which the programmer can tweak the toon shading algorithm with. (2 points)

9. Shadows (10 points)

a) Describe the shadow mapping algorithm using a sketch and a few explanatory sentences. (4 points)

b) Name two potential problems or artifacts that may appear with shadow mapping, and explain what can be done to fix them. (2 points)

c) Name one advantage and one disadvantage of shadow mapping compared to shadow volumes. (2 points)

d) Explain how the geometry for the shadow volume in the shadow volume algorithm is constructed. (2 points)
10. L-Systems (10 points)

The following L-system describes a variant of the Koch curve that uses only right angles. The L-system has a single variable F, two constants + and −, and the rule $F \rightarrow F+F−F−F+F$, where F means “draw forward”, + means “turn left ninety degrees”, and − means “turn right ninety degrees”. The starting sequence is F. Draw this curve for one and two levels of recursion.