CSE168
Computer Graphics II, Rendering

Spring 2006
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Final project

- Project description due Wednesday May 24
Last time

- Irradiance caching
Global illumination
Indirect irradiance
Indirect irradiance

[Humphreys, Pharr]
Irradiance caching

- Assume diffuse surfaces
- Cache irradiance samples instead of incident radiance as in photon mapping
- Interpolate cached samples
- Compute new samples only if interpolation fails
Irradiance caching algorithm

Three components

- Irradiance sampling
- Irradiance caching
- Irradiance interpolation

- Similar to photon mapping, but all steps are performed in main rendering pass
Irradiance sampling

- Assign a range for each sample, within which it can be used for interpolation
- Where irradiance changes quickly, range should be small
- Where irradiance changes slowly, range should be large
- Rate of change of irradiance depends on distance to visible surfaces
Irradiance sampling

[Image of a room with geometric objects lit by multi-colored lights]

[Wojciech Jarosz]
Irradiance caching

• Store samples in octree
• Add sample to each cell that it overlaps
• Adaptively subdivide octree such that each cell has limited number of samples
Irradiance caching examples

1000 sample rays, w>10
Irradiance caching examples

1000 sample rays, w>10

[Wann Jensen]
Irradiance caching examples

5000 sample rays, $w>10$  

[Wann Jensen]
Irradiance caching examples

5000 sample rays, w>10

[Wann Jensen]
Course recap

- Part 1: Implementing a basic ray tracer
- Part 2: Physics of light transport
- Part 3: Advanced topics
Course recap

Part I: Implementing a basic ray tracer

- Generating primary rays
- Ray-surface intersection
- Shading
- Acceleration structures
- Texturing
Course recap

Part 2: Physics of light transport

• Radiometry
• Reflection equation
• Monte Carlo integration
• Rendering equation
• Path tracing
• Photon mapping
• Irradiance caching
Outlook

Part 3: Advanced topics

- Sampling and aliasing
- Realistic camera models and HDR imaging
- Participating media and subsurface scattering
- Radiosity
- Guest lecture on visualization and VR
Today
Sampling and aliasing
  • Introduction
  • Fourier analysis
  • Antialiasing
Introduction

• Conceptually, an image is a continuous signal describing the radiance arriving at the image plane

• Signal is synonymous for function
Introduction

- Digital images are sampled representations of these continuous signals.
- Sampled means “defined only at discrete locations”, pixel centers.
Introduction

• Sampling: How to compute the value of the sampled pixel
• Reconstruction: How to obtain a continuous image from a set of samples
Aliasing occurs because of sampling and reconstruction.
Aliasing

Jagged boundaries
Aliasing

Improperly rendered detail
Aliasing

- Moire patterns
Aliasing

Sufficiently sampled

Insufficiently sampled

[R. Cook]
Sampling and aliasing

Is it possible to perfectly sample and reconstruct an image?

If yes, under what circumstances?
Fourier analysis

- All periodic signals can be represented as a summation of sinusoidal waves

[http://axion.physics.ubc.ca/341-02/fourier/fourier.html]
Fourier analysis

• The Fourier transform computes the amplitude and phase of the sinusoidal wave at each frequency

\[ F(\omega) = \int_{-\infty}^{\infty} f(x)e^{-i\omega x} \, dx \]

- Frequency \( \omega \)
- Complex amplitude \( F(\omega) \)
Fourier analysis

• Each signal can be represented in the spatial or the frequency domain

Fourier transform

\[ F(\omega) = \int_{-\infty}^{\infty} f(x)e^{-i\omega x} \, dx \]

Inverse Fourier transform

\[ f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)e^{i\omega x} \, d\omega \]
Fourier analysis

- Band-limited signal, no frequencies above a certain threshold

Spatial domain

Frequency domain, power spectrum
Fourier transform example

Spatial domain

\[
\text{square}(x) = \begin{cases} 
  1 & |x| \leq \frac{1}{2} \\
  0 & |x| > \frac{1}{2} 
\end{cases}
\]

Frequency domain

\[
\int_{-\infty}^{\infty} \text{square}(x) e^{-i\omega x} \, dx = \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-i\omega x} \, dx
\]

\[
= \left. \frac{e^{-i\omega x}}{-i\omega} \right|_{-\frac{1}{2}}^{\frac{1}{2}}
\]

\[
= e^{i\frac{1}{2}\omega} - e^{-i\frac{1}{2}\omega}
\]

\[
= \frac{2i}{\omega} \frac{\sin \frac{1}{2}\omega}{\frac{1}{2}\omega}
\]

\[
= \text{sinc}\, f
\]
Duality

$sinc \xleftarrow{F} \text{box}$

$\text{box} \xleftarrow{F} \text{sinc}$
Fourier transform example

**Spatial domain**

**Frequency domain**

**cosine (even)**

\[ \cos(-\omega t) = \cos(\omega t) \]

**sine (odd)**

\[ \sin(-\omega t) = -\sin(\omega t) \]
Dirac delta function

• Definition

\[ \delta(x) = \begin{cases} 
\infty & x = 0 \\
0 & x \neq 0 
\end{cases} \quad \int_{-\infty}^{\infty} \delta(x) \, dx = 1 \]

• Sifting property

\[ \int_{-\infty}^{\infty} f(x_0) \delta(x - x_0) = f(x_0) \]
Dirac delta function

Fourier transform

\[ \delta(x) \]
Impulse train

**Spatial domain**

\[ \text{III}_T(x) = \sum_k \delta(x - kT) \]

**Frequency domain**

\[ \text{III}_{\omega_0}(\omega) = \frac{1}{\sqrt{2\pi \omega_0}} \sum_k \delta(\omega - k\omega_0) \]

**Period**

- Period \( T \)
- Period \( \omega_0 = \frac{2\pi}{T} \)
Convolution

- In the spatial domain

\[ f(x) * g(x) = \int_{-\infty}^{\infty} f(t)g(x - t)dt = \tilde{f}(x) \]

Convolution kernel, filter \( g(x) \)
Filtered signal \( \tilde{f}(x) \)
Convolution

- In the spatial domain

\[ f(x) * g(x) = \int_{-\infty}^{\infty} f(t)g(x - t)dt = \tilde{f}(x) \]
Convolution

- In the spatial domain
  \[ f(x) \ast g(x) = \int_{-\infty}^{\infty} f(t)g(x - t) \, dt = \tilde{f}(x) \]

- Corresponds to multiplication in the spatial domain

- Convolution leads to the same result as Fourier transform, multiplication, inverse Fourier transform
Convolution

Spatial domain

\[ f(x) \quad g(x) \quad f(x) \ast g(x) \]

Frequency domain

\[ F(\omega) \quad G(\omega) \quad F(\omega) \cdot G(\omega) \]
Low-pass filters

- The larger the support in the spatial domain, the smaller the support in the frequency domain
High-pass filter
High-pass filter
Sampling

- Spatial domain: multiply signal with impulse train

\[ f(x) \rightarrow f(x) III_T(x) \]
Sampling

- Spatial domain: multiply signal with impulse train
  \[ f(x) \rightarrow f(x)III_T(x) \]

- Frequency domain: convolve signal with Fourier transform of impulse train
  \[ F(\omega) \rightarrow F(\omega) \ast III_{\omega_0}(\omega) \]
Sampling and reconstruction

Spatial Domain  Frequency Domain  Spatial Domain  Frequency Domain

continuous input  \(a_c(x)\)  \(A_c(\omega)\)  

sampling grid  \(I(x)\)  \(I_s(\omega)\)  reconstruction filter  \(\text{reconstructed signal (\textit{=original})}\)  

sampled signal  \(s(x)\)  \((s \ast \text{sinc})(x)\)  2

no aliasing  \(A(\omega)\)  \(A(\omega)B_s(\omega)\)
Sampling and reconstruction

Spatial Domain  Frequency Domain  Spatial Domain  Frequency Domain

Continuous input  \( a_c(x) \)  \( A_c(\omega) \)  \( \sin(x) \)  Reconstruction filter  \( B_3(\omega) \)

Sampling grid  \( i(x) \)  \( I(\omega) \)  \( (a*\sin)(x) \)  \( A(\omega)B_3(\omega) \)

Sampled signal  \( a(x) \)  \( A(\omega) \)  Aliasng  \( \text{Reconstructed signal (aliased)} \)
Sampling Theorem (Shannon 1949)

- A signal can be reconstructed exactly if it is sampled, at least, at twice its maximum frequency
- The minimum sampling frequency is called the Nyquist frequency
Anti-aliasing in graphics

Image signals are not band-limited to half the pixel frequency in general

- Prefiltering
- Supersampling
Prefiltering

- Band-limit the continuous signal to the Nyquist frequency before sampling
- Theoretically the way to go
Prefiltering

- Not applicable as a general solution in graphics, since continuous image signal is not known
- Useful for feature filtering (mip-mapping)
Supersampling

- Compute several intermediate samples per pixel
- Reconstruct final pixel sample from intermediate samples

Continuous pixel — Sample — Reconstruct — Sampled pixel
Supersampling

- Sampling patterns
- Reconstruction filters
Sampling patterns

• Two perspectives to assess quality of sampling patterns
  - Avoid aliasing
  - Efficient Monte Carlo integration

• Today: focus on aliasing
Uniform supersampling

- Increases Nyquist limit
- Spectrum of uniform sampling grid is also a uniform grid of Dirac impulses
- Aliases are coherent and very noticeable
- Can aliasing be made “less visually disturbing”? 
Distribution of extrafoveal cones

- Studied by Yellot (1983)
- Visual system is less sensitive to high frequency noise
Blue noise characteristics

- Least conspicuous form of aliasing is produced by sampling grids with two properties (*blue noise* properties)
  - Spectrum should be noisy and lack any concentrated spikes of energy
  - Spectrum should have a deficiency of low-frequency energy
- Aliasing is converted into broadband noise
- Noise is incoherent and less objectionable
Jittered Sampling

- Add uniform random jitter to each sample

Spatial domain  
Frequency domain

[Hanrahan]  
[Hanrahan]
Jittered vs. Uniform Supersampling

4x4 jittered sampling  
4x4 uniform sampling

[Hanrahan]  [Hanrahan]
Poisson Disk Sampling

- Random sampling with minimum distance constraint
- Dart throwing algorithm
Poisson Disk Sampling

2x2 Poisson sampling

2x2 uniform sampling

[Dippe 85]

[Dippe 85]
Reconstruction

- Reconstruction filters are weighting functions to compute a weighted average of the samples.
Box Filter

• Pretending pixels are little squares
• Take the average of samples in each pixel

\[
sinc(f_s/2) = \frac{\sin(\pi x)}{\pi x}
\]
Box filter

- Pixels are not little squares...

Original high-resolution image

Down-sampled with a 5x5 box filter (uniform weights)

Horizontal banding artifacts
The Ideal Reconstruction Filter

- Unfortunately it has *infinite* spatial extent
  - Every sample contributes to every interpolated point
- Expensive/impossible to compute
- Ringing (Gibbs phenomenon)
Problems with Reconstruction Filters

- Excessive pass-band attenuation results in blurry images
- Excessive high-frequency leakage can accentuate the sampling grid
- Filters with a small support in the spatial domain have a large support in the frequency domain
Mitchell-Netravali Filters [1988]

\[ k(x) = \frac{1}{6} \begin{cases} 
(12 - 9B - 6C) |x|^3 + & \text{if } |x| < 1 \\
(-18 + 12B + 6C) |x|^2 + (6 - 2B) & \text{if } 1 \leq |x| < 2 \\
(-B - 6C) |x|^3 + (6B + 30C) |x|^2 + & \text{otherwise} \\
(-12B - 48C) |x| + (8B + 24C) & 
\end{cases} \]
Reconstruction from non-uniform samples

- Requires normalization

\[ f(x) = \frac{\sum_{i=-\infty}^{\infty} f(x) k(x - x_i)}{\sum_{i=-\infty}^{\infty} k(x - x_i)} \]

Samples \( f(x_n) \)
Reconstruction kernel \( k(x) \)
Non-uniform positions \( x_n \)
Next time

- Realistic camera models and HDR imaging