CSE168
Computer Graphics II, Rendering

Spring 2006
Matthias Zwicker
Final project

Render an awe-inspiring image

- Soft shadows
- Path tracing
- Photon mapping
- Illumination from an environment map
- Subsurface scattering
- Motion blur, depth of field
- Procedural modeling, smoke, water
Final project

Previous years

• See web pages
Final project

Organization

• Work out project description, due next Wednesday May 24

• Groups up to 3 students possible, but need to define individual contributions explicitly for individual grading

• Project presentation Monday June 12, 7pm
Final project

Prizes

• First prize: $1000 contribution to a trip to SIGGRAPH

• Grand prize: $100 gift certificate for books

• Honorable mention: $50 gift certificate for books
Last time

- Explanation of cosine terms in photon transport
- Photon mapping rendering algorithms
Cosine terms

Path tracing

Photon tracing
Photon mapping algorithm

Recap

• Emit and transport photons
• Build data structure for fast access
• Use stored photons to estimate reflected radiance
Global illumination

100000 photons, 50 photons in radiance estimate
Visualization of the photons

[Wann Jensen]
Global illumination

500000 photons, 500 photons in radiance estimate
Photons for indirect illumination

10000 photons, 500 photons in radiance estimate

[Wann Jensen]
Splitting up the reflection equation

- **Reflection equation**
  \[ L_o(x, \omega_o) = \int_{H^2} f(x, \omega_o, \omega_i) L_i(x, \omega_i) \cos \theta_i d\omega_i \]

- **Split up BRDF**
  \[ f(x, \omega_o, \omega_i) = f_s(x, \omega_o, \omega_i) + f_d(x, \omega_o, \omega_i) \]

- **Split up incoming radiance**
  \[ L_i(x, \omega_i) = L_{i,l}(x, \omega_i) + L_{i,c}(x, \omega_i) + L_{i,d}(x, \omega_i) \]
Photon maps

• Three photon maps, one for each illumination term
  - Direct
  - Indirect diffuse
  - Caustic (indirect specular)

• Finite state machine to keep track of photon type
Direct illumination

• Sample light sources
Specular reflection

- Path tracing
Caustics

- Radiance estimation using caustic photon map
Reflections inside a ring

50000 photons, 50 photons for radiance estimation
Diffuse indirect illumination

• Path tracing, final gathering using all three photon maps, radiance estimation using indirect photons
Today

• Irradiance caching
• Radiosity
Diffuse indirect illumination

- Path tracing: slow
- Final gathering using all three photon maps: slow
- Radiance estimation using indirect photons: inaccurate for reasonable number of photons
- Other ideas?
Global illumination
Indirect irradiance

[Wann Jensen]
Indirect irradiance

[Humphreys, Pharr]
Indirect irradiance

- Changes very smoothly on diffuse surfaces, except for caustics
Irradiance caching

• “A ray tracing solution for diffuse interreflection”, Ward, Rubinstein, Clear, SIGGRAPH 88
• Assume diffuse surfaces
• Cache irradiance samples instead of incident radiance as in photon mapping
• Interpolate cached samples
• Compute new samples only if interpolation fails
Irradiance caching algorithm

Three components

• Irradiance sampling
• Irradiance caching
• Irradiance interpolation

• Similar to photon mapping, but all steps are performed in main rendering pass
Irradiance sampling

\[ E(\mathbf{x}) = \int_{\mathcal{H}^2} L_i(\mathbf{x}, \omega_i) \cos \theta_i d\omega_i \]
Irradiance sampling

\[ E(x) = \int_{\mathcal{H}^2} L_i(x, \omega_i) \cos \theta_i d\omega_i \]

\[ = \int_0^{2\pi} \int_0^{\pi/2} L_i(x, \theta_i, \phi) \cos \theta_i \sin \theta d\theta d\phi \]
Irradiance sampling

\[ E(\mathbf{x}) = \int_{\mathcal{H}^2} L_i(\mathbf{x}, \omega_i) \cos \theta_i d\omega_i \]

\[ = \int_0^{2\pi} \int_0^{\pi/2} L_i(\mathbf{x}, \theta_i, \phi) \cos \theta_i \sin \theta d\theta d\phi \]

\[ \approx \frac{\pi}{TP} \sum_{t=1}^{T} \sum_{p=1}^{P} L_i(\theta_t, \phi_p) \]

- Stratified sampling of the hemisphere
- Subdivision \( T,P \), uniform random variables \( \xi, \psi \)

\[ \theta_t = \sin^{-1} \left( \sqrt{\frac{t - \xi}{T}} \right), \text{ and } \phi_p = 2\pi \frac{p - \psi}{P} \]
Irradiance sampling

- Compute $L_i(x, \omega_i)$ using path tracing or photon gathering at the first hit point
- For good quality expect tracing 200-5000 paths
- Costly, but we will do this only at few locations in the image
Irradiance sampling

- Assign a range for each sample, within which it can be used for interpolation
- Where irradiance changes quickly, range should be small
- Where irradiance changes slowly, range should be large
- Rate of change of irradiance depends on distance to visible surfaces
Irradiance sampling

[Wojciech Jarosz]
Irradiance sampling

Harmonic mean heuristics

• The range is given by a radius

\[ r_j = \frac{N}{\sum_{i=1}^{N} 1/d_i} \]

where \( N \) is the number of paths, \( d_i \) is the distance to the first intersection along the path

• Average would weight infinite distances too heavily
Irradiance caching

Irradiance sample

```c
struct irradiance_sample {
    vector3 E   // irradiance
    vector3 n  // normal
    vector3 p  // position
    float r    // range
}
```
Irradiance caching

- Store samples in octree
- Add sample to each cell that it overlaps
- Adaptively subdivide octree such that each cell has limited number of samples
Irradiance caching

Octree example

• Adaptive subdivision such that each cell contains <3 samples
Irradiance interpolation

• Need to determine which samples should be used for interpolation

Error estimate

• Given a point on the surface \( x \) with normal \( n(x) \), estimate the difference to the irradiance cached at \( x_j \)
Irradiance interpolation

Error estimate

• Ad-hoc estimate for error of sample $j$

$$\epsilon_j(x) \leq E_j \left( \frac{4 \|x - x_j\|}{\pi r_j} + \sqrt{2 - 2n(x) \cdot n(x_j)} \right)$$
Irradiance interpolation

• Interpolation weights

\[ w_j(x) = \frac{1}{\|x-x_j\| + r_j \sqrt{1 - n(x) \cdot n(x_j)}} \approx \epsilon_j(x) \]

• Interpolated irradiance

\[ E(x) = \frac{\sum_i w_i(x) E(x_i)}{\sum_i w_i(x)} \]
Irradiance caching algorithm

\[ W = 0 \]
\[
\text{for( all irradiance samples } j \text{ in octree cell overlapping with } x ) \{ \\
\quad \text{compute weight } w_j \\
\quad \text{if( <sample is valid> ) } \{ \\
\quad \quad W += w_j; \ wE += w_j*E[j] \\
\quad \}\}
\]

\[
\text{if( } W > 0 \text{ ) } \{ \\
\quad \text{return } wE/W \\
\}\text{ else } \{ \\
\quad \text{return( compute new irradiance sample )} \\
\}\]
Irradiance caching algorithm

\texttt{<sample is valid>} =

\texttt{dist( x - x[j] ) < r[j]} \quad // \text{within range}
\texttt{&& w_j > 1/a} \quad // \text{sufficient weight}
\texttt{&& dot( x[j] - x, n(x) ) < 0} \quad // \text{x[j] is behind } x

Sample at \( x_j \)
\text{is invalid}
Non-diffuse surfaces

Approximation

\[ L_o(x, \omega_o) = \int_{\Omega^2} f(x, \omega_o, \omega_i) L_i(x, \omega_i) \cos \theta_i d\omega_i \]

\[ \approx \left( \int_{\Omega^2} f(x, \omega_o, \omega_i) d\omega_i \right) \left( \int_{\Omega^2} L_i(x, \omega_i) \cos \theta_i d\omega_i \right) \]
Non-diffuse surfaces

Approximation

\[ L_o(x, \omega_o) = \int_{H^2} f(x, \omega_o, \omega_i) L_i(x, \omega_i) \cos \theta_i d\omega_i \]

\[ \approx \left( \int_{H^2} f(x, \omega_o, \omega_i) d\omega_i \right) \left( \int_{H^2} L_i(x, \omega_i) \cos \theta_i d\omega_i \right) \]

\[ = \frac{1}{2} \rho_{hd}(\omega_o) E(x) \]

Hemispherical directional reflectance \( \rho_{hd}(\omega_o) \)
Photon mapping and irradiance caching

- Caustics break assumptions of irradiance caching
- Exclude caustic paths from irradiance sampling

Advanced technique

- Use photon map for importance sampling during path tracing
Irradiance caching examples

1000 sample rays, $w>10$
Irradiance caching examples

1000 sample rays, $w>10$

[Wann Jensen]
Irradiance caching examples

1000 sample rays, w>20

[Wann Jensen]
Irradiance caching examples

1000 sample rays, $w>20$
Irradiance caching examples

5000 sample rays, w>10

[Wann Jensen]
Irradiance caching examples

5000 sample rays, $w > 10$
Irradiance caching

[Humphreys, Pharr]
Irradiance caching example

[Humphreys, Pharr]
Path tracing

Approx. same amount of computation
Next time

- Midterm