CSE168
Computer Graphics II, Rendering

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Matthias Zwicker
Last time

- Path tracing and the measurement equation
- Photon mapping introduction
Measurement equation

- If we have random variable \( E[\tilde{L}_i] = L_i \) then we can compute any measurement

\[
I = \int \int W(x, \omega) L_i(x, \omega) d\omega dx
\]

\[
= E \left[ \frac{1}{N} \sum_j W(x_j, \omega_{i,j}) \tilde{L}_i(x_j, \omega_{i,j}) \right]
\]

for any importance function \( W \)

- Note: \( N \) is the number of paths
Path tracing

Bottom line: estimate pixel values as weighted average of random paths through pixel
Path tracing algorithm

- Construct paths incrementally starting at the eye, dozens of rays for each pixel
- Shoot shadow rays at each path vertex
- Compute pixel as weighted average of paths

[Cutler, Durand]
Problems with path tracing

1000 paths/pixel
Photon mapping

[Wann Jensen]
Photon mapping

• Photon emission and transport

[Cutler, Durand]
Photon mapping

- Photon caching

[Cutler, Durand]
Photon mapping

- Spatial data structure for fast access

[Cutler, Durand]
Photon mapping

- Radiance estimation

[Cutler, Durand]
What is a photon?

- A photon has a location and \( x \) a direction \( \omega_i \).
- A photon stores

\[
\alpha(x, \omega_i) = \frac{L_i^*(x, x, \omega_i)}{p(x, x, \omega_i)}
\]

such that the expected value is

\[
E[\alpha(x, \omega_i)] = L_i(x, \omega_i)
\]

class photon {
  vector3 x
  vector2 w_i
  vector3 a
}
Photon emission

Rectangular diffuse light

- Power $\Phi$
- Area $A$
- Emitted radiance $L_e(x, \omega) = \Phi/(2\pi A)$
- Choose uniform PDF $p(x, \omega) = 1/(2\pi A)$
- Initialize photon

$$\alpha_0 = \frac{L_e(x, \omega) \cos \theta}{2\pi A} = \Phi \cos \theta$$
Photon emission

- Store photon at first hit

\[
\text{photon\_cache->add(new photon( a\_0, x\_0, w\_i0 )}
\]
Photon transport

- Russian roulette with probability $q_j$ to abort after $j - 1$ bounces
- At each surface, choose random $\omega_{o,j}$ and update photon

$$\alpha_{j+1} = \alpha_j \frac{1}{1 - q_{j+1}} \frac{f(x_{j-1} \rightarrow x_j \rightarrow x_{j+1}) \cos \theta_j}{p(\omega_{o,j})}$$
Photon mapping

- Photon caching

$\alpha_j(x_j, \omega_{i,j})$

[Cutler, Durand]
Taking measurements

- Use the “average” incident radiance to estimate reflected radiance

\[ \alpha_j(x_j, \omega_{i,j}) \]

\[ \tilde{L}_o(x_c, \omega_o) \]
Taking measurements

- Measurement equation with “averaging kernel” $W$, total number of photons $N$

\[
\tilde{L}_o(x_c, \omega_o) = \int \int W(x, \omega)L_i(x, \omega)d\omega dx
\]

\[
= E \left[ \frac{1}{N} \sum_j W(x_j, \omega_{i,j})\alpha_i(x_j, \omega_{i,j}) \right]
\]

- Because we made sure that

\[
E[\alpha_j(x_j, \omega_{i,j})] = L_i(x_j, \omega_{i,j})
\]

- ...introduces bias
Taking measurements

• To measure reflected radiance, we choose
  \[ W(x_j, \omega_{i,j}) = \frac{1}{(\pi r^2)} f(x_c, \omega_{i,j}, \omega_o) \cos \theta_i \]
  for \( \|x - x_c\| < r \), 0 otherwise

• The estimate of the measurement is
  \[
  \tilde{L}_o(x_c, \omega_o) \approx \frac{1}{N \pi r^2} \sum_{j=1}^{k} f(x_c, \omega_{i,j}, \omega_o) \alpha_i(x_j, \omega_{i,j}) \cos \theta_i
  \]
  where \( \|x_j - x_c\| < r \) for all \( j \in 1 \ldots k \)

• \( N \) total paths/emitted photons
Global illumination

100000 photons, 50 photons in radiance estimate
Today

- Explanation of cosine terms in photon transport
- Photon mapping rendering algorithms
Example

- Measure flux through sensor surface.
Example

• Assume unit distances, constant BRDF $f$ and constant emitted radiance $L$

• Power $\Phi$

$$\Phi = \int_{A_M} \int_{\mathcal{H}^2(M)} L_i(x_m, \omega_m) \cos \theta_m d\omega_m dx_m$$
Example

- Assume unit distances, constant BRDF $f$ and constant emitted radiance $L$
- Power $\Phi$

\[
\Phi = \int_{A_M} \int_{H^2(M)} L_i(x_m, \omega_m) \cos \theta_m d\omega_m dx_m
\]

\[
= \int_{A_M} \int_{H^2(M)} \left( \int_{H^2(S)} f L \cos \theta_i d\omega_i \right) \cos \theta_m d\omega_m dx_m
\]
Example

- Assume unit distances, constant BRDF $f$ and constant emitted radiance $L$

- Power $\Phi$

$$\Phi = \int_{A_M} \int_{H^2(M)} L_i(x_m, \omega_m) \cos \theta_m \, d\omega_m \, dx_m$$

$$= \int_{A_M} \int_{H^2(M)} \left( \int_{H^2(S)} f L \cos \theta_i \, d\omega_i \right) \cos \theta_m \, d\omega_m \, dx_m$$

$$\approx \frac{1}{N} \sum \frac{f L \cos \theta_i \cos \theta_m}{p(x_m)p(\omega_m)p(\omega_i)}$$

path tracing
Example

\[ \Phi = \int_{A_M} \int_{\mathcal{H}^2(M)} L_i(x_m, \omega_m) \cos \theta_m \, d\omega_m \, dx_m \]

\[ = \int_{A_M} \int_{\mathcal{H}^2(M)} \left( \int_{\mathcal{H}^2(S)} f L \cos \theta_i \, d\omega_i \right) \cos \theta_m \, d\omega_m \, dx_m \]
Example

\[ \Phi = \int_{A_M} \int_{H^2(M)} L_i(x_m, \omega_m) \cos \theta_m d\omega_m dx_m \]

\[ = \int_{A_M} \int_{H^2(M)} \left( \int_{H^2(S)} f L \cos \theta_i d\omega_i \right) \cos \theta_m d\omega_m dx_m \]

\[ = \int_{A_M} \int_{A_S} \int_{A_L} f L \cos \theta_i \cos \theta_l dx_l \cos \theta_m \cos \theta_o dx_s dx_m \]

surface form, completely symmetric
\[ \Phi = \int_{A_M} \int_{\mathcal{H}^2(M)} L_i(x_m, \omega_m) \cos \theta_m d\omega_m dx_m \]

\[ = \int_{A_M} \int_{\mathcal{H}^2(M)} \left( \int_{\mathcal{H}^2(S)} f L \cos \theta_i d\omega_i \right) \cos \theta_m d\omega_m dx_m \]

\[ = \int_{A_M} \int_{A_S} \int_{A_L} f L \cos \theta_i \cos \theta_l dx_l \cos \theta_m \cos \theta_o dx_s dx_m \]

\[ = \int_{\mathcal{H}^2(S)} \int_{\mathcal{H}^2(L)} \int_{A_L} f L \cos \theta_l dx_l \cos \theta_o d\omega_l d\omega_o \]
Example

\[ \Phi = \int_{A_M} \int_{H^2(M)} L_i(x_m, \omega_m) \cos \theta_m d\omega_m dx_m \]

\[ = \int_{A_M} \int_{H^2(M)} \left( \int_{H^2(S)} f L \cos \theta_i d\omega_i \right) \cos \theta_m d\omega_m dx_m \]

\[ = \int_{A_M} \int_{A_S} \int_{A_L} f L \cos \theta_i \cos \theta_l dx_l \cos \theta_m \cos \theta_o dx_s dx_m \]

\[ = \int_{H^2(S)} \int_{H^2(L)} \int_{A_L} f L \cos \theta_l dx_l \cos \theta_o d\omega_l d\omega_o \]

\[ \approx \frac{1}{N} \sum \frac{f L \cos \theta_l \cos \theta_o}{p(x_l)p(\omega_l)p(\omega_o)} \quad \text{photon mapping} \]
Photon mapping algorithm

Recap
1. Emit and transport photons
2. Build data structure for fast access
3. Use stored photons to estimate radiance
Kd-tree for fast photon access

[Cutler, Durand]
Global illumination

100000 photons, 50 photons in radiance estimate

[Wann Jensen]
Visualization of the photons
Global illumination

500000 photons, 500 photons in radiance estimate
Photons for indirect illumination

10000 photons, 500 photons in radiance estimate
Splitting up the reflection equation

- Reflection equation
  \[ L_o(x, \omega_o) = \int_{H^2} f(x, \omega_o, \omega_i) L_i(x, \omega_i) \cos \theta_i d\omega_i \]

- Split up BRDF
  \[ f(x, \omega_o, \omega_i) = f_s(x, \omega_o, \omega_i) + f_d(x, \omega_o, \omega_i) \]

- Split up incoming radiance
  \[ L_i(x, \omega_i) = L_{i,l}(x, \omega_i) + L_{i,c}(x, \omega_i) + L_{i,d}(x, \omega_i) \]
Splitting up the reflection equation

- Reflection equation

\[
L_o(x, \omega) = \int_{H^2} f(x, \omega_0, \omega_i) L_i(x, \omega_i) \cos \theta_i d\omega_i
\]

\[
= \int_{H^2} f(x, \omega_0, \omega_i) L_{i,l}(x, \omega_i) \cos \theta_i d\omega_i
\]

\[
+ \int_{H^2} f_s(x, \omega_0, \omega_i)(L_{i,c}(x, \omega_i) + L_{i,d}(x, \omega_i)) \cos \theta_i d\omega_i
\]

\[
+ \int_{H^2} f_d(x, \omega_0, \omega_i)L_{i,c}(x, \omega_i) \cos \theta_i d\omega_i
\]

\[
+ \int_{H^2} f_d(x, \omega_0, \omega_i)L_{i,d}(x, \omega_i) \cos \theta_i d\omega_i
\]

- Evaluate each part separately, each with different algorithm
Photon maps

• Three photon maps, one for each illumination term
  - Direct
  - Indirect diffuse
  - Caustic (indirect specular)

• Finite state machine to keep track of photon type
Direct illumination
Direct illumination

\[ \int_{\mathcal{H}_2} f(x, \omega_o, \omega_i) L_{i,l}(x, \omega_i) \cos \theta_i d\omega_i \]

- Path tracing
- Sample light sources
- Multiple importance sampling
Specular reflection

[Wann Jensen]
Specular reflection

\[ \int_{h^2} f_s(x, \omega_o, \omega_i) (L_{i,c}(x, \omega_i) + L_{i,d}(x, \omega_i)) \cos \theta_i d\omega_i \]

- Path tracing
- No need to store photons on purely specular surfaces
Caustics
Caustics

\[ \int_{H^2} f_d(x, \omega_o, \omega_i) L_{i,c}(x, \omega_i) \cos \theta_i d\omega_i \]

- Indirect illumination due to purely specular paths (LS+D paths)
- Radiance estimation using caustic photon map
Diffuse indirect illumination

\[ \int_{\mathcal{H}^2} f_d(x, \omega_o, \omega_i) L_{i,d}(x, \omega_i) \cos \theta_i d\omega_i \]

- Indirect illumination that has bounced at least once off a diffuse surface
- Accurate: path tracing
- Less accurate, often good enough: final gathering using all three photon maps
- Even less accurate: radiance estimation using indirect photons
Diffuse indirect illumination

- Final gathering
Reflections inside a ring

50000 photons, 50 photons for radiance estimation
Caustics on glossy surfaces

340000 photons, ~100 photons for radiance estimation
HDR environment illumination

[Wann Jensen]

Lightprobe from www.debevec.org
Next time

- Radiosity