CSE168
Computer Graphics II, Rendering

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Last time

- Ray-surface intersection
- Implicit, parametric surfaces
- Spheres, triangles, polygons
- Shading
for all pixels {
    computeprimary( &ray )
    for all objects {
        intersect( ray, &hit )
        if hit is closer than firsthit {
            firsthit = hit
        }
    }
    shade( firsthit )
}
Ray-object intersection

- Ray \( p(t) = e + td \)
- Implicit surfaces \( f(p) = 0 \)
- Ray-surface intersection \( f(p(t)) = 0 \)
- Solve for \( t \)
- Surface normal \( n = \nabla f(p) \)
- Examples: sphere, infinite plane
Ray-object intersection

- Parametric surface
  \[x = f(u, v)\]
  \[y = g(u, v)\]
  \[z = h(u, v)\]

- Ray-surface intersection
  \[e_x + td_x = f(u, v)\]
  \[e_y + td_y = g(u, v)\]
  \[e_z + td_z = f(u, v)\]

- Solve for \(t, u, v\)
- Surface normal
  \[\mathbf{n}(u, v) = \left(\frac{\partial f}{\partial u}, \frac{\partial g}{\partial u}, \frac{\partial h}{\partial u}\right) \times \left(\frac{\partial f}{\partial v}, \frac{\partial g}{\partial v}, \frac{\partial h}{\partial v}\right)\]
- Example: triangles
Shading

- The BRDF (bi-directional reflectance distribution function) $\rho(l, v)$
- Fraction of light arriving from source that is transported towards viewer
Shading

- Phenomenological model

\[
\text{diffuse} + \text{specular} + \text{ambient} = \text{result}
\]
Shading

- Phenomenological models
  - “Hacks”, heuristics
- Other ways to define reflection models?
Shading

- Phenomenological models
  - “Hacks”, heuristics
- Other ways to define reflection models?
- Measurements
- Physical models
BRDF Measurement

- Gonioreflectometer

[Levoy]
Physical models

• What is light?
Physical models

- What is light?
  - Electromagnetic wave, [Maxwell 1862]
  - Made of photons, i.e., particles [Planck 1900]
  - Wave-particle duality [Einstein, early 1900]
    - “It depends on the experiment you are doing whether light behaves as particles or waves”
Physical models in computer graphics

- Large scale simulation requires approximations
- Geometric (ray) optics instead of wave optics in computer graphics
- Today we will also see some results based on wave optics
- Background, e.g, [http://en.wikipedia.org/wiki/Light](http://en.wikipedia.org/wiki/Light)
Today

• Reflection
• Refraction
• Shadows
Mirror reflection

\[ L(x, d) = R_s L_i(x, r) \]
\[ r = d - 2(d \cdot n)n \]
Mirror reflection

- Trace rays recursively to determine $L_i(x, r)$
- Limit recursion depth

```cpp
color mirror::shade( hit ) {
    ray = reflected_ray( hit )
    trace( ray, &mirror_hit )
    return Rs*shade( mirror_hit )
}
```
Mirror reflection
Specular refraction

- Diamond, glass, water, air
- Dielectric materials (insulators)
- Light travels at different speeds
- Light is bent when it goes from one medium to another
Specular refraction

• Fermat’s principle
  “The actual path between two points taken by a beam of light is the one which is traversed in the least time. “

• Index of refraction of a medium (dielectric material)

\[ n = \frac{\text{speed of light in vacuum}}{\text{speed of light in medium}} \]
Fermat’s principle
Snell’s law

\[ n \sin \theta = n_t \sin \phi \]
**Refracted direction**

- **Snell’s law**
  \[ n \sin \theta = n_t \sin \phi \]

- **Identity**
  \[ \sin^2 \theta + \cos^2 \theta = 1 \]

- **Using basis** \( b, n \) for plane of refraction
  \[ t = \sin \phi b - \cos \phi n \]
  \[ d = \sin \theta b - \cos \theta n \]

- **Eliminate** \( b \)
  \[ t = \sin \phi \frac{d + \cos \theta n}{\sin \theta} - \cos \phi n \]
Refracted direction

- Substitute

\[
\cos^2 \phi = 1 - \frac{n^2(1 - \cos^2 \theta)}{n_t^2}
\]

\[
\cos \theta = -d \cdot n
\]

\[
\sin \phi = \frac{n}{n_t} \sin \theta
\]

\[
t = \frac{n(d - n(d \cdot n))}{n_t} - n \sqrt{1 - \frac{n^2(1 - (d \cdot n)^2)}{n_t^2}}
\]
A glass sphere

- One light source
A glass sphere

- Two light sources
Dispersion

- Index of refraction varies with wavelength
Dispersion
Specular refraction

How much light is reflected and refracted?
Fresnel equations

- Derived from the electromagnetic wave equations
- Reflection and refraction from smooth, dielectric surfaces
- Involves polarization of the wave
Light waves

- Electromagnetic waves

Polarization

- Electric wave vector

[http://en.wikipedia.org/wiki/Polarization]
Polarization

- Natural light is only partially polarized
- Sources of highly polarized light
  - Direct sunlight
  - Reflections from dielectrics
- Polarization is diminished by multiple scattering
Fresnel equations

- Reflection of light polarized parallel and perpendicular to the plane of refraction

\[
\rho_\parallel = \frac{n_t \cos \theta - n \cos \phi}{n_t \cos \theta + n \cos \phi}
\]

\[
\rho_\perp = \frac{n \cos \theta - n_t \cos \phi}{n \cos \theta + n_t \cos \phi}
\]

\[
F_r(\theta) = \frac{1}{2} \left( \rho_\parallel^2 + \rho_\perp^2 \right)
\]

\[
F_t(\theta) = 1 - F_r(\theta)
\]
Fresnel equations

• Schlick approximation

\[ R(\theta) = R_0 + (1 - R_0)(1 - \cos \theta)^5 \]

• Reflectance at normal incidence

\[ R_0 = \left( \frac{n_t - 1}{n_t + 1} \right)^2 \]
Glass sphere

- No Fresnel reflection
Glass sphere

- With Fresnel reflection
Microfacet models

- Non-smooth surfaces
- Distribution of microfacet orientations
- Torrance-Sparrow model

Masking

Shadowing

Interreflection
Teapot series
Teapot series

[Image of a teapot]
Teapot series
Teapot series
Shadows

[Jensen]
Shadow rays

Occluder

Shadow ray
Shadow rays

[Jensen]
Shadow rays

Only accept intersections if $t > \epsilon$, $\epsilon > 0.00001$
Next time

- Acceleration structures
- Bounding volume hierarchies
- Paper on class web site